ISSN 2229-5518

Electron Density of States in High Temperature Superconductors

A.P. Singh

ABSTRACT
The evaluation of one electron thermodynamic Green's functions using the equation of motion technique of quantum dynamics via newly formulated Hamiltonian and using Dyson's equation approach. This involves approximation free approach and different cooper pairs are emerged automatically in the system and stands as an *ab-initio* approach. The detailed description of enhancement in the electron density of states (EDOS) for high temperature superconductors has been investigated. The investigated expressions of EDOS in the new framework are found responsible to describe a large number of dynamical properties of high temperature superconductors. The temperature dependence of EDOS has been found as a unique feature of the theory, which certainly becomes the outcome of the anharmonic interactions. The presence of electron-phonon interaction parameter in each term is an additional and new feature of the theory.

Index terms:- Anharmonicity, Cooper pairs, Electron Density of States, Green's function, Hamiltonian, Superconductors.

1 Introduction

Usually the potential energy of a crystal is expanded in terms of a Taylor series of nuclear displacements about their equilibrium positions. In this series the quadratic terms are known as the harmonic terms which yield the well known harmonic approximation and the rest containing higher-order (cubic-, quartic-, etc.) terms describe the anharmonic approximation [1]. The harmonic approximation however suffers from the problem of adjustable parameters could successfully explain a large number of physical properties but could not explain successfully many other physical properties of the crystals, such as the thermal expansion, thermal conductivity, infrared absorption, Raman scattering etc.

The harmonic approximation which treats the normal modes (exact eigen state of Schrödinger equation describing them infinitely long lived) as independent is an excellent approximation. If one includes the contribution of higher order terms in the expansion of energy, the Schrödinger equation does not offer any exact eigen state and the modes becomes short lived due to interaction of phonons and other excitations making the problem as many body problem.

In the early stage of development of many-body theory, it was extensively studied on the basis of perturbation theory [2-12]. Things now become much complicated but solved the problems amicably [13, 14]. Many of the more developments have occurred in treating the dynamical properties and have been studied with the use of the techniques of double-time thermodynamic Green functions [15, 16]: using the equation-of-motion technique of quantum dynamics, diagrammatic perturbation theory and functional derivatives [17, 18]. Further, the effect of anharmonicities does not vanish even at the Lowest of temperature [19].

Department Physics, M. M. College, Modinagar-201204 (UP), INDIA; Email;-dramanpal.singh23@gmail.com

Many refinements were incorporated in the theory of double-time temperature dependent Green's function applied to many body theory made it a very powerful method in solving many problems of condensed matter physics [20, 21]. The effects of anharmonicity on electrons and phonons make it a novel problem with immense academic interest for physicists due to: (a) harmonic crystals is an ideal situation which cannot be achieved in general and in case of high temperature superconductors, it is impossible due to the increased number of atoms per unit cell, which in turn, increases the possibility of interaction of particles and other energy excitations, thus, overruling the possibility of non-interacting modes with infinite lifetime; (b) the influence of anharmonicities does not vanish even at the absolute zero temperature: and (c) the impurity induction and the probability of invoking impurity modes and interference modes is very high in the T_c superconductors. The involvements of anharmonic and impurity effects reveal drastic changes in the EDOS. The development of the theory includes a general and newly formulated Hamiltonian which involves the contribution due to the unperturbed phonons-, unperturbed electrons-, electrons and phonons-, anharmonicity-, and defects (mass and force constant charges) [22]. Now the renormalized mode electron energy expressions are separated in terms of anharmonic, defect, interference thereof and electron phonon-coupling interactions. These quantities show strong dependence on temperature via Fermi and Planck's distribution functions. The low and high temperature limits EDOS have been investigated in the hydrodynamic and low temperature regimes.

2 Formulation

The electron density of states(EDOS) in the Lehman representation can be written as [23,24]

$$N_{(ep)}(\in) = -\sum_{q} \operatorname{Im} G^{(e)}(q,\in) \qquad (1)$$

ISSN 2229-5518

where $G^{(e)}(q,\in)$ is the one electron Green's function, $\in$ and $\mathbf{q}$ stand for the electron energy and electron wave vector, respectively.

3 The Hamiltonian

We consider a three-dimensional Bravais crystal with volume V containing N atoms such that, n lattice sites are occupied by the randomly distributed substitutional impurity atoms, each of mass M', while the rest $(N-n)$ lattice sites are filled by the host atoms, each of mass M. The number of impurity atoms n is very less in comparison to the host atoms $[n << (N-n)]$, so that for a low impurity concentration $(c = n/N)$ the impurity-impurity interaction would be ignored [25]. The introduction of impurities greatly modifies the force constants between host and impurity atoms along with the change of mass. In the present work the theory is restricted to the nearest neighbor force constants. The almost complete Hamiltonian of such a system can be written as

$$H = H_p + H_e + H_{ep} + H_A + H_D \tag{2}$$

where H_p is the unperturbed phonon Hamiltonian [26-28], H_e is the unperturbed electron Hamiltonian [29,30], H_{ep} is the electron-phonon Hamiltonian [31-33], H_A is the anharmonic Hamiltonian [34-36] and H_D is the defect Hamiltonian [25,36-39] arising due to the substitutional impurities respectively, which is given by

$$H_p = \sum_{\mathbf{k}} \frac{\in_k}{4}[A^*_{\mathbf{k}}A_{\mathbf{k}} + B^*_{\mathbf{k}}B_{\mathbf{k}}] \tag{2a}$$

$$H_e = \sum_{\mathbf{q}} \in_{\mathbf{q}} b^*_{\mathbf{q}} b_{\mathbf{q}}, \tag{2b}$$

$$H_{ep} = \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k}} b^*_{\mathbf{Q}} b_{\mathbf{q}} B_{\mathbf{k}}, \tag{2c}$$

$$H_A = \sum_{s\geq 3}\sum_{\mathbf{k}_1,..,\mathbf{k}_s} V_s(\mathbf{k}_1,\mathbf{k}_2,...,\mathbf{k}_s) A_{\mathbf{k}_1}A_{\mathbf{k}_2 ......} A_{\mathbf{k}_s} \tag{2d}$$

And

$$H_D = \sum_{\mathbf{k}_1,\mathbf{k}_2}[D(\mathbf{k}_1,\mathbf{k}_2)A_{\mathbf{k}_1}A_{\mathbf{k}_2} - C(\mathbf{k}_1,\mathbf{k}_2)B_{\mathbf{k}_1}B_{\mathbf{k}_2}] \tag{2e}$$

In above expressions $A_{\mathbf{k}} = a_{\mathbf{k}} + a^*_{-\mathbf{k}} = +A^*_{-\mathbf{k}}$ (phonon field operator) and $B_{\mathbf{k}} = a_{\mathbf{k}} - a^*_{-\mathbf{k}} = -B^*_{-\mathbf{k}}$ (phonon momentum operator), $b_{\mathbf{q}}(b^*_{\mathbf{q}})$ and $a_{\mathbf{k}}(a^*_{\mathbf{k}})$ are electron and phonon annihilation (creation) operators with wave vectors $\mathbf{q}$ and $\mathbf{k}$ respectively. For brevity and simplicity the subscripts have been redefined as $\mathbf{q}\sigma \equiv \mathbf{q}$ and $\mathbf{Q}\sigma \equiv \mathbf{Q}$ for electron and $\mathbf{k}(\equiv \mathbf{k}j)$ for phonon respectively, and $\mathbf{Q} = \mathbf{k} + \mathbf{q}$. $\in_{\mathbf{k}}, \in_{\mathbf{q}}, g_{\mathbf{k}}$, $V_s(\mathbf{k}_1,\mathbf{k}_2,...,\mathbf{k}_s)$, $C(\mathbf{k}_1,\mathbf{k}_2)$ and $D(\mathbf{k}_1,\mathbf{k}_2)$ stand for the electron energy, phonon energy and electron-phonon coupling coefficient, anharmonic coefficients, mass difference and force constant change parameters, respectively [25-37].

4 Electron Green's function

Let us consider the double-time thermodynamic electron retarded Green's functions

$$G_{qq'}(t,t') = \langle\langle b^*_{q\sigma}(t); b_{q'\sigma'}(t')\rangle\rangle = -i\theta(t-t') < [b^*_{q\sigma}, b_{q'\sigma'}] > \tag{3}$$

In above Eq.(3), σ defines the spin and $\uparrow(\downarrow)$ designates the spin up (down) for electrons.

With the help of equation of motion technique of quantum dynamics via Hamiltonian (2) in the form [36]

$$G_{qq'}(\in) = \frac{(3\in^N + \in^C)\delta_{qq'}\delta_{\sigma\sigma'}}{\{2\pi[\in^2 - \tilde{\in}^2_{q\sigma} + (3\in^N + \in^C)P(q,\in)]\}} \tag{4}$$

where $\tilde{\in}^2_{q\sigma}$ $(\equiv \tilde{\in}^2_q)$ is the renormalized energy, which is equal to

$$\tilde{\in}^2_q = (3\in^N + \in^C)^2 - \frac{X_1}{2\pi} - \frac{1}{(3\in^N + \in^C)}\sum_{q'}(g_k + g^*_{k'})\frac{X_2}{2\pi} \tag{5}$$

and

$$P(q,\in) = \frac{1}{2\pi(3\in^N + \in^C)^2}\langle\langle F^*_{q\sigma}(t); F_{q'\sigma'}(t')\rangle\rangle_{\in} \tag{6}$$

where $\in^C$ is the energy of cooper pair. In Eq.(4) the delta function $\delta_{q\sigma}$ acquires a large number of momentum and spin combinations, namely $\delta_{q\uparrow}\delta_{q\uparrow}$, $\delta_{q\uparrow}\delta_{q\downarrow}$, $\delta_{q\downarrow}\delta_{q\uparrow}$, $\delta_{q\downarrow}\delta_{q\downarrow}$, $\delta_{-q\uparrow}\delta_{q\uparrow}$, $\delta_{-q\uparrow}\delta_{q\downarrow}$, $\delta_{-q\downarrow}\delta_{q\uparrow}$, $\delta_{-q\downarrow}\delta_{q\downarrow}$. During the above development it is surprisingly found that the cooper pair energy $\in^C$ as well as the normal electron energy $\in^N$ automatically emerges out in the results. The solution of function $P(q,\in)$ can be obtained after decoupling of the developed Green's functions from the Green's function $\langle\langle F^*_{q\sigma}(t); F_{q'\sigma'}(t')\rangle\rangle$, with the help of electron and phonon renormalized Hamiltonians

$$H_{ren(e)}^{(0)} = \sum_q (\tilde{\in}_{q\uparrow}\, b_{q\uparrow}^* b_{q\uparrow} + \tilde{\in}_{q\downarrow}\, b_{q\downarrow}^* b_{q\downarrow} + \tilde{\in}_{-q\uparrow}\, b_{-q\uparrow}^* b_{-q\uparrow} + \tilde{\in}_{-q\downarrow}\, b_{-q\downarrow}^* b_{-q\downarrow}) \tag{7}$$

$$H_{ren(p)}^{(0)} = \frac{1}{4}\sum_k [\frac{\tilde{\in}_k^2}{\in_k} A_k^* A_k + \in_k\, B_k^* B_k] \tag{8}$$

As

$$\begin{aligned} P(q,\in) = \sum_{k,k'} G_{k,k'}^2 \Bigg\{ & \Bigg[\left(-\frac{8\tilde{\in}_k^2}{\in_k} + \frac{2\in_k^3}{(3\in^N + \in^C)^2}\right)\frac{N_{Q\sigma}}{\in^2 - \tilde{\in}_k^2} + \left(\frac{\in_k^2\, n_k}{(3\in^N + \in^C)^2} + \frac{4\in_k\, \tilde{n}_k}{3\in^N + \in^C} + \tilde{\tilde{n}}_k\right]\frac{4}{\in - (3\tilde{\in}^N + \tilde{\in}^C)} \\ & + 128\sum_{k_1}\left[D(k_1,-k)D(-k_1,-k')\frac{\in_{k_1} N_{Q\sigma}}{\in^2 - \tilde{\in}_{k_1}^2} + D(k_1,-k)D(k_1,-k')\frac{n_{k_1}}{\in - (3\tilde{\in}^N + \tilde{\in}^C)}\right]\frac{1}{(3\in^N + \in^C)^2} \\ & + 288\sum_{k_1,k_2}\Bigg[V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k')\left(\frac{S_{+\alpha}\, \tilde{\in}_{+\alpha}}{\in^2 - \tilde{\in}_{+\alpha}^2} + \frac{S_{-\alpha}\, \tilde{\in}_{-\alpha}}{\in^2 - \tilde{\in}_{-\alpha}^2}\right)\eta_1 N_{Q\sigma} \\ & + V_3(k_1,k_2,-k)V_3(k_1,k_2,-k')\left(\frac{n_{k_1} n_{k_2}}{\in - (3\tilde{\in}^N + \tilde{\in}^C)}\right)\Bigg]\frac{1}{(3\in^N + \in^C)^2} \\ & + 1536\sum_{k_1,k_2,k_3}\Bigg[V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k')\left(\frac{S_{+\beta}\, \tilde{\in}_{+\beta}}{\in^2 - \tilde{\in}_{+\beta}^2} + 3\frac{S_{-\beta}\, \tilde{\in}_{-\beta}}{\in^2 - \tilde{\in}_{-\beta}^2}\right)\eta_2 N_{Q\sigma} \\ & + V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k')\left(\frac{3n_{k_1} n_{k_2} n_{k_3}}{\in - (3\tilde{\in}^N + \tilde{\in}^C)}\right)\Bigg]\frac{1}{(3\in^N + \in^C)^2}\Bigg\} \end{aligned} \tag{9}$$

where

$$n_k = \langle A_k A_k \rangle ;\ \tilde{n}_k = \langle A_k B_k \rangle ;\ \tilde{\tilde{n}}_k = \langle B_k B_k \rangle \tag{9a}$$

$$G_{k,k'}^2 = (g_k + g_k^*)(g_{k'} + g_{k'}^*) \tag{9b}$$

$$\tilde{\in}_{\pm\alpha} = \tilde{\in}_{k_1} \pm \tilde{\in}_{k_2} ;\ \tilde{\in}_{\pm\beta} = \tilde{\in}_{k_1} \pm \tilde{\in}_{k_2} \pm \tilde{\in}_{k_2} ; \tag{9c}$$

$$S_{\pm\alpha} = n_{k_2} \pm n_{k_1} ;\ S_{\pm\beta} = 1 \pm n_{k_1} n_{k_2} + n_{k_2} n_{k_3} \pm n_{k_3} n_{k_1} \tag{9d}$$

$$\eta_1 = \frac{\in_{k_1}\in_{k_2}}{\tilde{\in}_{k_1}\tilde{\in}_{k_2}} ;\ \eta_2 = \frac{\in_{k_1}\in_{k_2}\in_{k3}}{\tilde{\in}_{k_1}\tilde{\in}_{k_2}\tilde{\in}_{k_3}} \tag{9e}$$

and

$$N_{Q\sigma} = \int_{-\infty}^{+\infty} \frac{e^{-i\in(t-t')}}{e^{\beta\in} \pm 1}\delta[\in - (3\tilde{\in}^N + \tilde{\in}^C)]d\in \tag{9f}$$

The value of $N_{Q\sigma}$ is including through the cooper pairs and normal electron problem via $\delta[\in - (3\in^N + \in^C)]$ function. This cannot be normally solved with usual process. Hence, here we can write above expressions as (to the reasonable degree of accuracy and without violating physical lows)

$$N_{Q\sigma} \cong \int_{-\infty}^{+\infty} e^{-i\in(t-t')}\frac{1}{2}\left[\frac{\delta(\in - 3\in^N)}{e^{\beta\in} + 1} + \frac{\delta(\in - \in^C)}{e^{\beta\in} - 1}\right]d\in \tag{10}$$

or

$$N_{Q\sigma} \cong \frac{1}{2}\left[\frac{1}{e^{3\beta\in^N} + 1} + \frac{1}{e^{\beta\in^C} - 1}\right]\ (t = t') \tag{11}$$

The electron-phonon energy shifts $\Delta_{(ep)}(q,\in)$ are the principal value of $P(q,\in)$. The shift $\Delta_{(ep)}(q,\in)$ and line width $\Gamma_{(ep)}(q,\in)$ can be separated in three terms, such as

$$\Delta_{(ep)}^{EP}(q,\in) = 4\sum_{k,k'} G_{k,k'}^2 \left\{\left[-\frac{8\tilde{\in}_k^2}{\in_k} + \frac{2\in_k^3}{(3\in^N + \in^C)^2}\right]\frac{N_{Q\sigma}}{\in^2 - \tilde{\in}_k^2} + \left[\frac{\in_k^2\, n_k}{(3\in^N + \in^C)^2} + \frac{4\in_k\, \tilde{n}_k}{(3\in^N + \in^C)} + \tilde{\tilde{n}}_k\right]\frac{1}{\in - (3\tilde{\in}^N + \tilde{\in}^C)}\right\} \tag{12}$$

$$\Delta_{(ep)}^{D}(q,\in) = 128\sum_{\substack{k,k' \\ k',k_1}} G_{k,k'}^2 \left\{D(k_1,-k)D(-k_1,-k')\frac{\in_{k_1} N_{Q\sigma}}{\in^2 - \tilde{\in}_{k_1}^2} + D(k_1,-k)D(k_1,-k')\left[\frac{n_{k_1}}{\in - (3\tilde{\in}^N + \tilde{\in}^C)}\right]\right\}\frac{1}{(3\in^N + \in^C)^2} \tag{13}$$

$$\Delta^{3A}_{(ep)}(q,\in) = 288\sum_{\substack{k,k',\\k_1,k_2}} G^2_{k,k'}\left\{V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k')\eta_1 N_{Q\sigma}\left[\frac{S_{+\alpha}\ \tilde{\in}_{+\alpha}}{\in^2-\tilde{\in}^2_{+\alpha}}+\frac{S_{-\alpha}\ \tilde{\in}_{-\alpha}}{(\in^2-\tilde{\in}^2_{-\alpha})}\right]\right.$$
$$\left.+V_3(k_1,k_2,-k)V_3(k_1,k_2,-k')\left[\frac{n_{k_1}n_{k_2}}{\in-(3\tilde{\in}^N+\tilde{\in}^C)}\right]\right\}\frac{1}{(3\in^N+\in^C)^2} \tag{14}$$

$$\Delta^{4A}_{(ep)}(q,\in) = 1536\sum_{\substack{k,k',k_1,\\k_2,k_3}} G^2_{k,k'}\left[V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k')\left[\frac{S_{+\beta}\ \tilde{\in}_{+\beta}}{\in^2-\tilde{\in}^2_{+\beta}}+3\frac{S_{-\beta}\ \tilde{\in}_{-\beta}}{\in^2-\tilde{\in}^2_{-\beta}}\right]\eta_2 N_{Q\sigma}\right.$$
$$\left.+V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k')\left[\frac{3n_{k_1}n_{k_2}n_{k_3}}{\in-(3\tilde{\in}^N+\tilde{\in}^C)}\right]\right\}\frac{1}{(3\in^N+\in^C)^2} \tag{15}$$

In above Eqs.(12), (13), (14)and (15), the second term of each part of defect-, anharmonic- and phonon-electron terms shows the renormalize cooper pair and normal energy which denoted by $\tilde{\in}^C$ and $\tilde{\in}^N$ respectively. Now, the electron line width $\Gamma_{(ep)}(q,\in)$ can be written as

$$\Gamma_{(ep)}(q,\in) = \Gamma^{EP}_{(ep)}(q,\in)+\Gamma^{D}_{(ep)}(q,\in)+\Gamma^{3A}_{(ep)}(q,\in)+\Gamma^{4A}_{(ep)}(q,\in) \tag{16}$$

where

$$\Gamma^{EP}_{(ep)}(q,\in) = 4\pi\sum_{k,k'} G^2_{k,k'}\left\{\xi(\in)\left[-\frac{8\tilde{\in}^2_k}{\in_k}+\frac{2\in^3_k}{(3\in^N+\in^C)^2}\right]N_{Q\sigma}\delta(\in^2-\tilde{\in}^2_k)\right.$$
$$\left.+\left[\frac{\in^2_k\ n_k}{(3\in^N+\in^C)^2}+\frac{4\in_k\ \tilde{n}_k}{(3\in^N+\in^C)}+\tilde{\tilde{n}}_k\right]\delta[\in-(3\tilde{\in}^N+\tilde{\in}^C)]\right\} \tag{17}$$

$$\Gamma^{D}_{(ep)}(q,\in) = 128\pi\sum_{\substack{k,k'\\k',k_1}} G^2_{k,k'}\{\xi(\in)D(k_1,-k)D(-k_1,-k')\in_{k_1}\ N_{Q\sigma}\delta(\in^2-\tilde{\in}^2_{k_1})$$
$$+D(k_1,-k)D(k_1,-k')n_{k_1}\delta[\in-(3\tilde{\in}^N+\tilde{\in}^C)]\}\frac{1}{(3\in^N+\in^C)^2} \tag{18}$$

$$\Gamma^{3A}_{(ep)}(q,\in) = 288\pi\sum_{\substack{k,k',\\k_1,k_2}} G^2_{k,k'}\{V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k')\xi(\in)\eta_1 N_{Q\sigma}[S_{+\alpha}\ \tilde{\in}_{+\alpha}\delta(\in^2-\tilde{\in}^2_{+\alpha})$$
$$+S_{-\alpha}\ \tilde{\in}_{-\alpha}\ \delta(\in^2-\tilde{\in}^2_{-\alpha})]+V_3(k_1,k_2,-k)V_3(k_1,k_2,-k')n_{k_1}n_{k_2}\delta[\in-(3\tilde{\in}^N+\tilde{\in}^C)]\}\frac{1}{(3\in^N+\in^C)^2} \tag{19}$$

$$\Gamma^{4A}_{(ep)}(q,\in) = 1536\pi\sum_{\substack{k,k',k_1,\\k_2,k_3}} G^2_{k,k'}\{V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k')\xi(\in)\eta_2 N_{Q\sigma}$$
$$\times[S_{+\beta}\ \tilde{\in}_{+\beta}\ \delta(\in^2-\tilde{\in}^2_{+\beta})+3S_{-\beta}\ \tilde{\in}_{-\beta}\ \delta(\in^2-\tilde{\in}^2_{-\beta})]$$
$$+3V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k')n_{k_1}n_{k_2}n_{k_3}\delta[\in-(3\tilde{\in}^N+\tilde{\in}^C)]\} \tag{20}$$

5 Electron Density of States (EDOS)

Now, the imaginary part of $G^{(e)}(q,\in)$ is given by

$$G^{(e)}(q,\in) = \frac{(3\in^N+\in^C)^2\delta_{qq'}\delta_{\sigma\sigma'}\Gamma_{(ep)}(q,\in)}{\{2\pi[(\in^2-\overline{\in}^2_q)^2+(3\in^N+\in^C)^2[\Gamma_{(ep)}(q,\in)]^2]\}} \tag{21}$$

where the energy of perturbed mode is given by

$$\overline{\in}^2_q = \tilde{\in}^2_q + (3\in^N+\in^C)\Delta(q,\in) \tag{22}$$

Using Eq.(21) in (1), we obtain the EDOS as

$$N_{(ep)}(\in) = \frac{(3\in^N + \in^C)^2 \Gamma_{(ep)}(q,\in)}{\{2\pi[(\in^2 - \overline{\in}_q^2)^2 + (3\in^N + \in^C)^2[\Gamma_{(ep)}(q,\in)]^2]\}} \quad (23)$$

this can be reasonably approximated for small value of line width in the form

$$N_{(ep)}(\in) = \sum_i N_{(ep)}^i(\in) = \sum_i \frac{(3\in^N + \in^C)^2 \Gamma_{(ep)}^i(q,\in)}{[2\pi(\in^2 - \overline{\in}_q^2)^2]} \quad (24)$$

$$i = EP, 3A, 4A \text{ and } D$$

The various contributions to the EDOS can be summarized as:

$$N_{(ep)}^{EP}(\in) = 2\Omega_v \sum_{k,k'} G_{k,k'}^2 \left\{ \left[-4\,\tilde{\in}_k^3 \in_k^{-1} (3\in^N + \in^C)^2 + \in_k^3 \tilde{\in}_k \right] \frac{1}{(\tilde{\in}_k^2 - \overline{\in}_q^2)^2} N_{Q\sigma} + \left[\frac{\in_k^2 n_k}{2} + 2\in_k (3\in^N + \in^C)\tilde{n}_k + 2(3\in^N + \in^C)^2 \tilde{\tilde{n}}_k \right] \frac{(3\tilde{\in}^N + \tilde{\in}^C)^2}{[(3\tilde{\in}^N + \tilde{\in}^C)^2 - \overline{\in}_q^2]^2} \right\} \quad (25)$$

$$N_{(ep)}^{D}(\in) = 32\Omega_v \sum_{\substack{k,k' \\ k',k_1}} G_{k,k'}^2 \left\{ \left[D(k_1,-k)D(-k_1,-k')\,\tilde{\in}_{k_1} \in_{k_1} \right] \frac{1}{(\tilde{\in}_{k_1}^2 - \overline{\in}_q^2)^2} N_{Q\sigma} + D(k_1,-k)D(k_1,-k') n_{k_1} \frac{(3\tilde{\in}^N + \tilde{\in}^C)^2}{[(3\tilde{\in}^N + \tilde{\in}^C)^2 - \overline{\in}_q^2]^2} \right\} \quad (26)$$

$$N_{(ep)}^{3A}(\in) = 72\Omega_v \sum_{\substack{k,k', \\ k_1,k_2}} G_{k,k'}^2 \left\{ V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k') \left[\frac{S_{+\alpha}\,\tilde{\in}_{+\alpha}^2}{(\tilde{\in}_{+\alpha}^2 - \overline{\in}_q^2)^2} + \frac{S_{-\alpha}\,\tilde{\in}_{-\alpha}^2}{(\tilde{\in}_{-\alpha}^2 - \overline{\in}_q^2)^2} \right] \eta_1 N_{Q\sigma} + V_3(k_1,k_2,-k)V_3(k_1,k_2,-k') \frac{(3\tilde{\in}^N + \tilde{\in}^C)^2}{(3\tilde{\in}^N + \tilde{\in}^C)^2 - \overline{\in}_q^2} n_{k_1} n_{k_2} \right\} \quad (27)$$

$$N_{(ep)}^{4A}(\in) = 384\Omega_v \sum_{\substack{k,k', \\ k_1,k_2,k_3}} G_{k,k'}^2 \left\{ V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k') \left[\frac{S_{+\beta}\,\tilde{\in}_{+\beta}^2}{(\tilde{\in}_{+\beta}^2 - \overline{\in}_q^2)^2} + 3\frac{S_{-\beta}\,\tilde{\in}_{-\beta}^2}{(\tilde{\in}_{-\beta}^2 - \overline{\in}_q^2)^2} \right] \eta_2 N_{Q\sigma} + V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k') 3 \frac{n_{k_1} n_{k_2} n_{k_3} (3\tilde{\in}^N + \tilde{\in}^C)^2}{[(3\tilde{\in}^N + \tilde{\in}^C)^2 - \overline{\in}_q^2]} \right\} \quad (28)$$

6 Density of States for Superconductors

When in the close vicinity of T_c the superconductivity increases so that the cooper pairs dominate over normal pairs. In this case $(\in^C >> \in^N)$ and thus,

$$N_{(ep)}^{EP}(k,C) = 2\Omega_v \sum_{k.k'} G_{k,k'}^2 \left\{ \frac{1}{2} \left[-\frac{4\,\tilde{\in}_k^3 (\in^C)^2}{\in_k} + \in_k^3 \tilde{\in}_k \right] \frac{n(\in^C)}{(\in^C)^4} \left[1 - \frac{[\tilde{\in}^2 (k,C) + \tilde{\in}_k^2]}{(\in^C)^2} \right] + \frac{n_k \in_k^2}{2(\tilde{\in}^C)^2} \left[1 + \frac{2[(\in^C)^2 + \tilde{\in}^2 (k,C)]}{(\tilde{\in}^C)^2} \right] \right\} \quad (29)$$

$$N_{(ep)}^{D}(k,C) = 32\Omega_v \sum_{\substack{k,k' \\ k',k_1}} G_{k,k'}^2 \left| D(k_1,-k) \right|^2 \left\{ \frac{\tilde{\in}_{k_1} \in_{k_1}}{2(\in^C)^4} \left[1 - \frac{[\tilde{\in}^2 (k,C) + \tilde{\in}_{k_1}^2]}{(\in^C)^2} \right] n(\in^C) + \frac{n_{k_1}}{(\tilde{\in}^C)^2} \left[1 + \frac{2[(\in^C)^2 + \tilde{\in}^2 (k,C)]}{(\tilde{\in}^C)^2} \right] \right\} \quad (30)$$

$$N_{(ep)}^{3A}(k.C) = 72\Omega_v \sum_{\substack{k,k',\\k_1,k_2}} G_{k,k'}^2 \left|V_3(k_1,k_2,-k)\right|^2 \left\{ \frac{\eta_1}{2(\in^C)^4} \left[\left(1 - \frac{[\widetilde{\in}^2\ (k,C) + \widetilde{\in}_{+\alpha}^2]}{(\in^C)^2} \right) \right.\right.$$

$$\left.\left. \times S_{+\alpha}\ \widetilde{\in}_{+\alpha}^2 + S_{-\alpha}\ \widetilde{\in}_{-\alpha}^2 \left(1 - \frac{[\widetilde{\in}^2\ (k,C) + \widetilde{\in}_{-\alpha}^2]}{(\in^C)^2} \right) \right] n(\in^C) + \frac{n_{k_1} n_{k_2}}{(\widetilde{\in}^C)^2} \left[1 + \frac{2[(\in^C)^2 + \widetilde{\in}^2\ (k,C)]}{(\widetilde{\in}^C)^2} \right] \right\} \quad (31)$$

$$N_{(ep)}^{4A}(k,C) = 384\Omega_v \sum_{\substack{k,k',\\k_1,k_2,k_3}} G_{k,k'}^2 \left|V_4(k_1,k_2,k_3,-k)\right|^2 \left\{ \eta_2 \left[\left(1 - \frac{[\widetilde{\in}^2\ (k,C) + \widetilde{\in}_{+\beta}^2]}{(\in^C)^2} \right) S_{+\beta}\ \widetilde{\in}_{+\beta}^2 \right.\right.$$

$$\left.\left. + 3S_{-\beta}\ \widetilde{\in}_{-\beta}^2 \left(1 - \frac{[\widetilde{\in}^2\ (k,C) + \widetilde{\in}_{-\beta}^2]}{(\in^C)^2} \right) \right] \frac{n(\in^C)}{2(\in^C)^4} + 3\frac{n_{k_1} n_{k_2} n_{k_3}}{(\widetilde{\in}^C)^2} \left[1 + \frac{2[(\in^C)^2 + \widetilde{\in}^2\ (k,C)]}{(\widetilde{\in}^C)^2} \right] \right\} \quad (32)$$

where to a reasonable approximation with vanishingly small value of $\tilde{\tilde{n}}_k$ and $\tilde{n}_k$ use here

$$\widetilde{\in}^2\ (k,C) \cong (\in^C)\Delta(k,C) - 12\sum_{k,k_1} (g_k + g_k^*) V_3(k_1,k_1,-k) n_{k_1} \quad (33)$$

with

$$\Delta(k,C) = \Delta_{(ep)}^{EP}(k,C) + \Delta_{(ep)}^{D}(k,C) + \Delta_{(ep)}^{3A}(k,C) + \Delta_{(ep)}^{4A}(k,C) \quad (34)$$

$$\Delta_{(ep)}^{EP}(k,C) = 4\sum_{k.k'} G_{k,k'}^2 \left[\left(-\frac{8\,\widetilde{\in}_k^2}{\in_k} + \frac{2\in_k^3}{(\in^C)^2} \right) \frac{n(\in^C)}{2(\in^2 - \widetilde{\in}_k^2)} + \left(\frac{\in_k^2}{(\in^C)^2} \right) \frac{n_k}{(\in - \widetilde{\in}^C)} \right] \quad (35)$$

$$\Delta_{(ep)}^{D}(k,C) = 128\sum_{\substack{k,k'\\k',k_1}} G_{k,k'}^2 \left|D(k_1,-k)\right|^2 \left[\frac{\in_{k_1} n_k}{2(\in^2 - \widetilde{\in}_{k_1}^2)} + \frac{n_{k_1}}{(\in - \widetilde{\in}^C)} \right] \frac{1}{(\in^C)^2} \quad (36)$$

$$\Delta_{(ep)}^{3A}(k,C) = 288\sum_{\substack{k,k',\\k_1,k_2}} G_{k,k'}^2 \left|V_3(k_1,k_2,-k)\right|^2 \left\{ \frac{1}{2}\left[\frac{S_{+\alpha}\ \widetilde{\in}_{+\alpha}}{(\in^2 - \widetilde{\in}_{+\alpha}^2)} + \frac{S_{-\alpha}\ \widetilde{\in}_{-\alpha}}{(\in^2 - \widetilde{\in}_{-\alpha}^2)} \right] \eta_1 n(\in^C) + \frac{n_{k_1} n_{k_2}}{(\in - \widetilde{\in}^C)} \right\} \frac{1}{(\in^C)^2} \quad (37)$$

$$\Delta_{(ep)}^{4A}(k,C) = 1536\sum_{\substack{k,k',k_1,\\k_2,k_3}} G_{k,k'}^2 \left|V_4(k_1,k_2,k_3,-k)\right|^2 \left\{ \frac{1}{2}\left[\frac{S_{+\beta}\ \widetilde{\in}_{+\beta}}{(\in^2 - \widetilde{\in}_{+\beta}^2)} + 3\frac{S_{-\beta}\ \widetilde{\in}_{-\beta}}{(\in^2 - \widetilde{\in}_{-\beta}^2)} \right] \eta_2 n(\in^C) + \frac{3n_{k_1} n_{k_2} n_{k_3}}{[\in - \widetilde{\in}^C)]} \right\} \frac{1}{(\in^C)^2} \quad (38)$$

In the case of superconductors the contribution of normal energy becomes least than the cooper pairs energy, so that the cooper pairs energy is greater than the normal energy $(\in^N < \in^C)$. Therefore, the contributions of EDOS can be written in this form

$$N_{(ep)}^{EP}(k,q) = 2\Omega_v \sum_{k,k'} G_{k,k'}^2 \left\{ \frac{1}{2}\left[-\frac{4\,\widetilde{\in}_k^3\ [(\in^C)^2 + 6\in^C\in^N]}{\in_k} + \in_k^3 \widetilde{\in}_k \right] \frac{n(\in^C)}{(\in^C)^4} \right.$$

$$\left. \times \left[1 - \frac{[\widetilde{\in}^2\ (k,q) + \widetilde{\in}_k^2]}{(\in^C)^2} \right] + \left[1 + \frac{2[(\in^C)^2 + 6\in^C\in^N + \widetilde{\in}^2\ (k,q)]}{(\widetilde{\in}^C)^2} \right] \frac{n_k \in_k^2}{2(\widetilde{\in}^C)^2} \right\} \quad (39)$$

$$N_{(ep)}^{D}(k,q) = 32\Omega_v \sum_{\substack{k,k'\\k',k_1}} G_{k,k'}^2 \left|D(k_1,-k)\right|^2 \left\{ \frac{\widetilde{\in}_{k_1}\in_{k_1}}{2(\in^C)^4} \left[1 - \frac{[\widetilde{\in}^2\ (k,q) + \widetilde{\in}_{k_1}^2]}{(\in^C)^2} \right] n(\in^C) \right.$$

$$\left. + \frac{n_{k_1}}{(\widetilde{\in}^C)^2} \left[1 + \frac{2[(\in^C)^2 + 6\in^C\in^N + \widetilde{\in}^2\ (k,q)]}{(\widetilde{\in}^C)^2} \right] \right\} \quad (40)$$

$$N_{(ep)}^{3A}(k.q)=72\Omega_v\sum_{\substack{k,k',\\k_1,k_2}}G_{k,k'}^2\left|V_3(k_1,k_2,-k)\right|^2\left\{\frac{\eta_1}{2(\in^C)^4}\left[\left(1-\frac{[\tilde{\in}^2(k,q)+\tilde{\in}_{+\alpha}^2]}{(\in^C)^2}\right)S_{+\alpha}\tilde{\in}_{+\alpha}^2+S_{-\alpha}\tilde{\in}_{-\alpha}^2\right.\right.$$

$$\left.\left.\times\left(1-\frac{[\tilde{\in}^2(k,q)+\tilde{\in}_{-\alpha}^2]}{(\in^C)^2}\right)\right]n(\in^C)+\frac{n_{k_1}n_{k_2}}{(\tilde{\in}^C)^2}\left[1+\frac{2[(\in^C)^2+6\in^C\in^N+\tilde{\in}^2(k,q)]}{(\tilde{\in}^C)^2}\right]\right\} \quad (41)$$

$$N_{(ep)}^{4A}(k,q)=384\Omega_v\sum_{\substack{k,k',\\k_1,k_2,k_3}}G_{k,k'}^2\left|V_4(k_1,k_2,k_3,-k)\right|^2\left\{\eta_2\left[\left(1-\frac{[\tilde{\in}^2(k,q)+\tilde{\in}_{+\beta}^2]}{(\in^C)^2}\right)S_{+\beta}\tilde{\in}_{+\beta}^2+3S_{-\beta}\tilde{\in}_{-\beta}^2\right.\right.$$

$$\left.\left.\times\left(1-\frac{[\tilde{\in}^2(k,q)+\tilde{\in}_{-\beta}^2]}{(\in^C)^2}\right)\right]\frac{n(\in^C)}{2(\in^C)^4}+3\frac{n_{k_1}n_{k_2}n_{k_3}}{(\tilde{\in}^C)^2}\left[1+\frac{2[(\in^C)^2+6\in^C\in^N+\tilde{\in}^2(k,C)]}{(\tilde{\in}^C)^2}\right]\right\} \quad (42)$$

where

$$\tilde{\in}^2(k,q)\cong(3\in^N+\in^C)\Delta(\in,q)-12\sum_{k,k_1}(g_k+g_k^*)V_3(k_1,k_1,-k)n_{k_1} \quad (43)$$

7 Discussion of Electron Density of States

The renormalized mode energy $\tilde{\in}_q$ can be evaluated in this form

$$\tilde{\in}_q^2=(3\in^N+\in^C)^2-4\sum_k(g_k+g_k^*)\left[\sum_{k'}(g_{k'}+g_{k'}^*)\tilde{\tilde{n}}_k+3\sum_{k_1}V_3(k_1,k_1,-k)n_{k_1}\right]-4\sum_k(g_k+g_k^*)$$

$$\times\sum_{k'}(g_{k'}+g_{k'}^*)\left[2(3\in^N+\in^C)\tilde{\tilde{n}}_k+\in_k\tilde{n}_k+24\sum_{k_1}V_4(k_1,k_1,k,-k)n_{k_1}\tilde{n}_k+4D(k,-k)\tilde{n}_k\right]\frac{1}{(3\in^N+\in^C)} \quad (44)$$

The Eq.(44) includes with the terms of anharmonic nature and those of an impurity anharmonicity interaction nature. These all terms shows strong temperature dependence via n_{k_1}, n_k, $\tilde{n}_k$ and $\tilde{\tilde{n}}_k$. The low (*l*) and high (*h*) temperature contributions to EDOS can be described as

$$N_{(ep)}^{EP}(\in,T)_l=2\Omega_v\sum_{k,k'}G_{k,k'}^2\{[-4\tilde{\in}_k^3\in_k^{-1}(3\in^N+\in^C)^2+\in_k^3\tilde{\in}_k(3\in^N+\in^C)]D(\tilde{\in}_k,\overline{\in}_q)\tilde{N}(\in^N,\in^C)$$

$$+[(\in_k^2S)/2+2\tilde{\in}_k\in_k^{-1}(3\in^N+\in^C)S+2\tilde{\in}_k^2\in_k^{-2}(3\in^N+\in^C)^2S]D[(3\tilde{\in}^N+\tilde{\in}^C),\overline{\in}_q](3\tilde{\in}^N+\tilde{\in}^C)^2\} \quad (45)$$

$$N_{(ep)}^{D}(\in,T)_l=32\Omega_v\sum_{\substack{k,k'\\k',k_1}}G_{k,k'}^2\{D(k_1,-k)D(-k_1,-k')\tilde{\in}_{k_1}\in_{k_1}D(\tilde{\in}_k,\overline{\in}_q)\tilde{N}(\in^N,\in^C)+D(k_1,-k)D(k_1,-k')$$

$$\times(3\tilde{\in}^N+\tilde{\in}^C)^2S_1D[(3\tilde{\in}^N+\tilde{\in}^C),\overline{\in}_q]\} \quad (46)$$

$$N_{(ep)}^{3A}(\in,T)_l=72\Omega_v\sum_{\substack{k,k',\\k_1,k_2}}G_{k,k'}^2\{V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k')\eta_1[S_+^{(1)}\tilde{\in}_{+\alpha}^2D(\tilde{\in}_{+\alpha},\overline{\in}_q)+S_-^{(1)}\tilde{\in}_{-\alpha}^2D(\tilde{\in}_{-\alpha},\overline{\in}_q)]$$

$$\times\tilde{N}(\in^N,\in^C)+V_3(k_1,k_2,-k)V_3(k_1,k_2,-k')(3\tilde{\in}^N+\tilde{\in}^C)^2D[(3\tilde{\in}^N+\tilde{\in}^C),\overline{\in}_q]S_1S_2\} \quad (47)$$

$$N_{(ep)}^{4A}(\in,T)_l=384\Omega_v\sum_{\substack{k,k',\\k_1,k_2,k_3}}G_{k,k'}^2\{V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k')\eta_2[S_+^{(2)}\tilde{\in}_{+\beta}^2D(\tilde{\in}_{+\beta},\overline{\in}_q)+3S_-^{(2)}\tilde{\in}_{-\beta}^2$$

$$\times D(\tilde{\in}_{-\beta},\overline{\in}_q)]\tilde{N}(\in^N,\in^C)+V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k')(3\tilde{\in}^N+\tilde{\in}^C)^2$$

$$\times D[(3\tilde{\in}^N+\tilde{\in}^C),\overline{\in}_q]S_1S_2S_3\} \quad (48)$$

$$N^{EP}_{(ep)}(\in,T)_h = 2\Omega_v \sum_k G^2_{k,k'} \{[-4\,\tilde{\in}^3_k \in^{-1}_k (3\in^N + \in^C)^2 + \in^3_k \tilde{\in}_k (3\in^N + \in^C)] D(\tilde{\in}_k, \overline{\in}_q)[(2\beta \in^C)^{-1} - (3\beta \in^N)/8] + \beta^{-1}[\in^3_k \tilde{\in}^{-2}_k + 4(3\in^N + \in^C) \in_k \tilde{\in}^{-1}_k + 4 \in^{-1}_k (3\in^N + \in^C)^2] \times D[(3\tilde{\in}^N + \tilde{\in}^C), \overline{\in}_q](3\tilde{\in}^N + \tilde{\in}^C)^2\} \quad (49)$$

$$N^D_{(ep)}(\in,T)_h = 32\Omega_v \sum_{\substack{k,k' \\ k',k_1}} G^2_{k,k'} \{D(k_1,-k)D(-k_1,-k')\,\tilde{\in}_{k_1} \in_{k_1} D(\tilde{\in}_k, \overline{\in}_q)[(2\beta \in^C)^{-1} - (3\beta \in^N)/8] + D(k_1,-k)D(k_1,-k')(3\tilde{\in}^N + \tilde{\in}^C)^2 \beta^{-1}\Omega_1 D[(3\tilde{\in}^N + \tilde{\in}^C), \overline{\in}_q]\} \quad (50)$$

$$N^{3A}_{(ep)}(\in,T)_h = 72\Omega_v \sum_{\substack{k,k', \\ k_1,k_2}} G^2_{k,k'} \{V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k')\eta_1[\Omega^{(1)}_+ \tilde{\in}^2_{+\alpha} D(\tilde{\in}_{+\alpha}, \overline{\in}_q) + \Omega^{(1)}_- \tilde{\in}^2_{-\alpha} D(\tilde{\in}_{-\alpha}, \overline{\in}_q)] \times [\frac{1}{(2\beta \in^C)} - \frac{(3\beta \in^N)}{8}] + V_3(k_1,k_2,-k)V_3(k_1,k_2,-k')(3\tilde{\in}^N + \tilde{\in}^C)^2 D[(3\tilde{\in}^N + \tilde{\in}^C), \overline{\in}_q]\frac{\Omega_1\Omega_2}{\beta^2}\} \quad (51)$$

$$N^{4A}_{(ep)}(\in,T)_h = 384\Omega_v \sum_{\substack{k,k', \\ k_1,k_2,k_3}} G^2_{k,k'} \{V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k')\eta_2[\Omega^{(2)}_+ \tilde{\in}^2_{+\beta} D(\tilde{\in}_{+\beta}, \overline{\in}_q) + 3\Omega^{(2)}_- \tilde{\in}^2_{-\beta} D(\tilde{\in}_{-\beta}, \overline{\in}_q)][\frac{1}{(2\beta \in^C)} - \frac{(3\beta \in^N)}{8}] + 3V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k') \times (3\tilde{\in}^N + \tilde{\in}^C)^2 D[(3\tilde{\in}^N + \tilde{\in}^C), \overline{\in}_q]\beta^{-3}\Omega_1\Omega_2\,\Omega_3\} \quad (52)$$

where

$$[\tilde{N}(3\in^N) + \tilde{N}(\in^C)] = \tilde{N}(\in^N, \in^C) \quad (53)$$

The EDOS is worth promising that $N_{(ep)}(\in,T)$ is a function of $\tilde{\in}^2_q$ which it depends on the anharmonic and defect contributions. These contributions show that the strong temperature dependence via $n_{k_1}, n_k, \tilde{n}_k$ and $\tilde{\tilde{n}}_k$. The cooper pairs play an important role in the expressions of EDOS for high temperature superconductors. In the close vicinity of Tc the superconductivity increases so that the cooper pairs dominate over normal pairs. Therefore, the contribution of normal energy becomes small compared to the cooper pair energy. Obviously, the evaluation of electron Green's function is in the heart of the problem, which can lead to describe the EDOS and various dynamical properties of a superconducting system.

Acknowledgement One of the authors Aman Pal Singh (Assistent Professor) Multanimal Modi College, Modinagar (UP) India.

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